Indian Statistical Institute
Chennai Centre
M.Stat. S.Stream I Year 2015-2016
Second Semester
Mid-Semester Examination

Design of Experiments 06.04.16

Answer as much as you can. The maximum you can score is 56.
The notation used have their usual meaning unless stated otherwise.
Time :- 2 hours

1. Consider a CRD with $v$ treatments with replication numbers $r_1, \cdots r_v$.

   (a) Write down the model in the form $Y = X_0\mu + X_1\tau + \varepsilon$, describing the meaning of each symbol.

   (b) Let $r = \begin{bmatrix} r_1 \\ \vdots \\ r_v \end{bmatrix}$ and $D_R = Diag(r_1, \cdots r_v)$.

   Show that $X_1'X_0 = r$ and $X_1'X_1 = D_R$.

   (c) Show that the least square estimate of $\tau$ can be obtained from $C_0\hat{\tau} = Q_0$, where $C_0 = D_R - (1/n)rr'$ and $Q_0 = T - (G/n)r$, $G$ is the grand total.

   $[1 + 3 + 5 = 10.]$

2. Consider a latin square design using a latin square of order $v$. Consider the following function $d$ from $V \times V$ to $V$ as follows. $d(i,j) = k$ if treatment $k$ is applied to the $(i,j)$th cell of the latin square. Let $S_k = \{(i,j) : d(i,j) = k\}$. Show that

   $\sum_{i=1}^{v}\sum_{j=1}^{v} Y_{ij}T_{d(i,j)} = \sum_{k=1}^{v} T_k^2$, where $Y_{ij}$ is the yield from the $(i,j)$th cell and $T_k$ is the total from the $k$th treatment. [3]

3. Consider a general block design with $v$ treatments with replications $r_1, \cdots r_v$, $b$ blocks of sizes $k_1, \cdots k_b$

   (a) Write down the model in the form $Y = X_0\mu + X_1\tau + X_2\beta + \varepsilon$, describing the meaning of each symbol.

   (b) Suppose the reduced normal equation for the treatment effects is $C\hat{\tau} = Q$. Express $C$ and $Q$ in terms of $X_i$'s and $Y$.[Proof is not needed].

   (c) Obtain $E(Q)$ and $Cov(Q)$.

   (d) Show that $l'\tau$ is estimable if and only if $l$ is in the column space of $C$.

   (e) Show that if $l'\tau$ and $m'\tau$ are estimable then $Cov(l'\hat{\tau}, m'\hat{\tau}) = \sigma^2 l'c^- m$.

   (f) Is $Cov(\hat{\tau})$ is $\sigma^2 c^-$ ? Justify. $[3 + 3 + (2 + 3) + 9 + 5 + 3 = 26]$

4. Consider $C$ in Q 3. Show the following.

   (a) $C$ is non-negative definite.

   (b) $C1_v = 0$.

   (c) If $\text{rank}(C) = v - 1$, then for any positive $a$, $E = C + aJ_v$ is positive definite and $E^{-1}$ is a g-inverse of $C$. [2 + 2 + 5 = 9]

5. (a)Consider the block design

   | | | | |
   |---|---|---|---|
   | Block 1 : | 1 | 1 | 2 |
   | Block 2 : | 3 | 3 | 4 |
   | Block 3 : | 2 | 4 | 5 |

   Is $\tau_1 - \tau_3$ estimable ? Justify

   (b)Consider the block design

   | | | |
   |---|---|---|
   | Block 1 : | 1 | 2 |
   | Block 2 : | 2 | 3 |
   | Block 3 : | 3 | 1 |

   (i) Obtain the $C$-matrix.

   (ii) Obtain an expression for the BLUE of an elementary treatment contrast in terms of treatment totals and block totals. [3 + (2 + 5) = 10]

6. Construct a BIBD with parameters $v = b = 13, r = k = 4, \lambda = 1$. [Either present all the blocks or describe the construction with proof]. [8]